Weekly Homework 1

Math 485

September 12, 2013

1. If two people are randomly chosen from a group of eight women and six men, what is the probability that

a. Both are women;

b. Both are men;

c. one is a man and the other a woman?

2. If A and B are independent, show that so are

a. A and B^c;

b. A^c and B^c.

3. Bayes' rule in probability states that

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

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a. Prove Bayes' rule using the definition of conditional probability.

b. The chance of contracting a rare disease X is .03. However, the diagnosis for X is not very reliable. A person who contracted X has probability .9 of being diagnosed positive. A person who does not contract X has probability of .05 of being diagnosed positive. Using Baye's rule, find the probability that a person who was diagnosed positive did **not** contract X.

4. Prove the following identities $\big($ You can take the linearity of Expectation: $E(aX + bY) = aE(X) + bE(Y)$ as given $\big)$

a. $E\big[(X - EX)^2\big] = E(X^2) - (EX)^2$.

b. $Cov(aX + bY, Z) = aCov(X, Z) + bCov(Y, Z)$.
c. $Cov(X, c) = 0, c$ a constant.
d. $Cov(X, Y) = 0$ if X, Y are independent.
e. $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$.
f. If X, Y are independent, $Var(X + Y) = Var(X) + Var(Y)$.

5. A club has 120 members, of whom 35 play chess, 58 play bridge and 27 play both chess and bridge. If a member of the club is randomly chosen, what is the conditional probability that the person

a. plays chess given that he or she plays bridge;
b. plays bridge given that he or she plays chess?

6. Frequent fliers of a certain airline fly a random number of miles each year, having a mean of 25,000 and standard deviation of 12,000 miles. If 30 such people are randomly chosen, approximate the probability that the average of their mileages for this year will
a. exceed 25,000.
b. be between 23,000 and 27,000.

7. An urn has 4 red balls and 5 blue balls. Three balls are drawn without replacement.

a. What is the probability that all of them are blue?
b. (Optional) Compute the probabilty that the third ball is blue (assuming the balls are drawn sequentially without replacement). Is it equal to the probability that the first ball is blue? Are you surprised by the result?

8. Suppose we have a coin with probability p of showing up Head. Let X be the number of coin toss until we see the first Head. Let Y be the result of the first toss, i.e. $Y = 1$ if the first toss shows head, 0 otherwise. Using the definitions given in class : $P(X = i|Y = j) = \frac{P(x=i,Y=j)}{P(Y=j)}$ etc., find the following:
a. $P(X = k|Y = 1)$ (What are the possible values of k here?)
b. $P(X = k|Y = 0)$ (What are the possible values of k here?)
c. $E(X|Y = 1)$.

d. $E(X|Y=0)$ - Here you can use the fact that the series $p+2(1-p)p+3(1-p)^2p+4(1-p)^3p+...$ is equal to $\frac{1}{p}$.

(Your results above should coincide with what we have obtained.)

9. We have showed the forward price $F(0,T)$ for a forward contract of a stock S with and expiration date T is $F(0,T) = S_0e^{rT}$. Suppose $F(0,T) < S_0e^{rT}$. Explain how the buyer (or the seller) would take advantage of this opportunity to guarantee a positive profit at time T. Repeat for the case $F(0,T) > S_0e^{rT}$.

10. A stock evolves according to the following model:

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\begin{aligned}
S_0 &= 10; \\
S_{i+1} &= S_i X_i.
\end{aligned}
$$

where X_i are iid random variables with distribution

$$
\begin{aligned}
X_i &= 1.5 \text{ with probability } 0.4 \\
X_i &= 0.7 \text{ with probabiltiy } 0.6.
\end{aligned}
$$

Compute $E(S_6|S_4)$. (This can be interpreted as the best guess for stock price on the 6th day given the stock price on the 4th day).